Frictional Labor Market, Spatial Sorting and Disparities

Jiong Wu*

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Abstract

This paper explores how frictional labor markets contribute to spatial labor sorting and, consequently, to disparities in productivity, wages, and unemployment across regions. The model incorporates frictional labor matching with two worker types, two locations, and free labor mobility. It predicts that skilled workers tend to sort into areas with higher productivity, higher wages, and lower unemployment rates. Empirical evidence aligns with these theoretical predictions, suggesting that frictional labor markets play a crucial role in shaping spatial economic disparities.

1 Introduction

Spatial disparities in key economic variables like productivity, wage, and unemployment are of great policy concern and academic interest (e.g., Ehrlich and Overman (2020)). Beneath the spatial disparities lies spatial sorting. It is an important factor in explaining not just the distributions of those economic variables but also the city size. This paper links spatial sorting with spatial differentials in productivity, wage, and unemployment through a new channel, frictional labor market, to answer the question: how can frictional labor market explain spatial sorting, hence disparities?

To address this question, I first present a static search model, based on an extension of Acemoglu (1999), with two locations and free labor mobility. The model considers two types of workers—skilled and unskilled—and firms that make ex-ante capital investment decisions before hiring. In a frictional labor market, firms and workers cannot freely change partners after meeting, bargaining, and forming a match. Firms increase investment only when the

*Department of Economics, University of Virginia (email: jw3tz@virginia.edu). I am grateful to Kerem Coşar, James Harrigan and John McLaren for their advice. I would also like to thank Zach Bethune and Eric Young for their comments. All errors are my own.

human capital difference between skilled and unskilled workers is sufficiently large and there is a high probability of encountering a skilled worker. Skilled workers then sort into locations offering higher wages due to increased firm investment, which further encourages firms to hire only skilled workers, deterring unskilled workers from entering. Unskilled workers, consequently, settle in the other location, accepting lower wages. The paper also identifies conditions under which a symmetric allocation of workers across locations can exist as an equilibrium.

The model is then extended to a dynamic setting, incorporating more general features of search models that address unemployment differences. The core insights from the static model persist, with the dynamic model predicting lower unemployment rates in areas with a higher concentration of skilled workers. This is because these areas attract more firms, increasing job-finding rates and reducing unemployment. Spatial sorting is usually explained by city size due to urban agglomeration, making it challenging to separate the two both theoretically and empirically (Combes and Gobillon (2015)). However, the theory presented here predicts sorting independently of total city size. The resulting equilibrium suggests that observed firm sorting may arise from variations in regional human capital levels rather than inherent differences in firm productivity.

I test the prediction at the commuting zone level using Census/ACS data. The results indicate that a higher fraction of skilled workers is positively associated with regional average wages and job-finding rates, and negatively associated with unemployment rates. The job finding rates for each commuting zone are measured indirectly, as neither the 5%-sample Census nor the ACS provide explicit information on individuals' lagged employment statuses. I exploit the answers to a question from these survey data to obtain proxies for the individuals' lagged employment statuses.

This paper contributes to the literature on spatial labor sorting. In the empirical literature, Andersson et al. (2007) found that larger urban labor markets exhibit more assortative matching between workers and firms, using U.S. data. This finding aligns with the predictions of my model, assuming a constant-return-to-scale matching function. Mion and Naticchioni (2009) employed matched employer-employee data from Italy to demonstrate that skills are geographically sorted, accounting for a significant share of spatial wage variation. Similarly, Matano and Naticchioni (2012), using the same dataset, showed that spatial sorting is not uniform across sectors. This finding supports my model's prediction that differences in production structures can lead to varying levels of sorting. Combes et al. (2008), using French panel data, concluded that skill-based spatial sorting explains a substantial portion of wage inequality and that differences in worker human capital across cities account for 40-50% of the size-productivity relationship. These empirical results inform the model developed in this paper, which further introduces the novel insight that production structure plays a key role in generating sorting. To the best of my knowledge, no prior studies have highlighted this mechanism.

The theoretical foundations of spatial sorting are often linked to urban agglomeration, making it challenging to separate sorting effects from agglomeration both theoretically and empirically (Combes and Gobillon (2015)). Behrens and Behrens and Robert-Nicoud (2015) empirically demonstrated that the proportion of skilled workers in a metropolitan statistical area (MSA) is positively correlated with the area's size and density. They extended Henderson's (1974) model to explain sorting through agglomeration externalities. Similarly, the theoretical frameworks of Davis and Dingel (2019, 2020) attribute spatial sorting of talent to agglomeration driven by costly idea exchanges within cities, once again linking sorting to city size. In contrast, my model predicts sorting without relying on agglomeration or city size. Diamond (2016) documented the spatial sorting of skilled workers in the U.S., noting that college graduates tend to cluster in high-wage, high-rent cities. She attributed this sorting to local labor productivity shocks. The increased skill sorting, driven by changes in labor demand, was further reinforced by endogenous improvements in amenities in these cities. Tabuchi et al. (2018) also used productivity shocks to explain regional disparities. Behrens et al. (2014) integrated sorting, selection, and agglomeration into a unified model, where sorting is driven by selection—tougher competition in larger cities results in more talented individuals remaining there. This concentration of talent, in turn, intensifies selection, leading firms to offer higher wages. The resulting wage premium from sorting and selection attracts more individuals, further reinforcing agglomeration economies. Eeckhout et al. (2014) found that average skill levels remain constant across cities of different sizes, as large cities disproportionately attract both high- and low-skilled workers. This finding challenges the theories that consistently link agglomeration to skill sorting: how can sorting occur if average skill levels do not vary with city size? The authors argued that complementarities between high- and low-skilled workers shape the distribution of skills within a city and influence how it varies by size. While my model does not address this "thicker tails" phenomenon, as it does not assume complementarities between different worker types, it does incorporate unemployment—an aspect that few spatial sorting models address.

This paper contributes to the literature on wage inequality. Extensive research documents a significant rise in wage inequality in the United States, attributing it primarily to skill-biased technological change (see Acemoglu and Autor (2011)). Autor and Dorn (2013) observed faster growth at both ends of the wage distribution between 1980 and 2005, attributing the rise in wage inequality to the declining costs of automating middle-skill jobs. In contrast, Moretti (2013) provided evidence that real wage inequality has grown less significantly than nominal wage differences. However, real wages may not fully capture well-being, as local amenities vary considerably across cities. Moretti argued that well-being inequality depends largely on why college graduates choose to reside in expensive metropolitan areas, with relative labor demand shocks playing a more critical role than labor supply factors. The model in this paper explains skilled-unskilled wage inequality through sorting. In the absence of sorting, firms would pool jobs and wages, leading skilled and unskilled workers to have similar job opportunities and wages, thereby eliminating inequality.

This paper contributes to the literature on spatial unemployment. Several studies have examined spatial unemployment differentials. OECD (2005) documented that these differentials are significant and persistent. Kline and Moretti (2013) and Marinescu and Rathelot (2018) focused on the role of the job-finding rate while abstracting from job-loss rate differentials. In contrast, Bilal (2023) found that gaps in job-loss rates are the key empirical determinant of spatial unemployment differentials, based on detailed data from France. This distinction has important policy implications. Kline and Moretti (2013) argued that subsidies to high-unemployment areas reduce welfare, while Bilal found that such subsidies can increase welfare, thereby reconciling theoretical models with real-world place-based policies. Although Bilal's analysis provides valuable insights into regional unemployment differences, the approach to modeling endogenous job separation lacks sufficient empirical support. The key feature of Bilal's model is a stochastic decay in firm productivity, which allows for the possibility of firm exit and varies across firms due to its stochastic nature. However, assuming such a decaying process for firm productivity is not entirely convincing, as other studies suggest that productivity may increase over time, such as through learning by exporting. This paper aligns with the literature that emphasizes the job-finding rate as a driver of unemployment differences. It also predicts that areas with lower unemployment rates are those where firms sort and have higher productivity, consistent with Bilal's findings.

2 Theory

I extend Acemoglu (1999) model to include two locations with free mobility, allowing for the endogenization of the proportions of skilled workers. This approach demonstrates how sorting among different types of workers can arise from a frictional labor market. I begin with a static version of the model to illustrate the core mechanism. The dynamic version introduces greater complexity, incorporating general features of a search model that addresses unemployment, including endogenous job-finding and vacancy contact rates, which were treated as exogenous in Acemoglu's original model.

2.1 Static model

There is exogenous heterogeneity in worker skill levels: some workers are unskilled with human capital normalized to 1, while others are skilled with a human capital level of η. By distinguishing between different types of workers, agglomeration is characterized by the concentration of each worker type. Let $\bar{\phi}$ represent the exogenous fraction of skilled workers in the total labor force, which is inherent in the economy. The labor market clearing conditions are as follows:

$$L_1^H + L_2^H = \bar{\phi} L,$$
$$L_1^L + L_2^L = (1 - \bar{\phi}) L,$$

where H and L denote the skilled and unskilled workers. I further denote ϕ_i as the fraction of skilled workers in location i: $\phi_i = L_i^H / (L_i^H + L_i^L)$, which is the key variable as shown below. Notice that ϕ_1 and ϕ_2 are endogenously determined.

The timeline of the static model begins with a firm deciding on the level of physical capital to allocate to a potential worker. However, the firm must make this decision before meeting the worker and knowing their type. In a frictional labor market, it is assumed that each worker meets only one firm, and each firm meets only one worker, randomly. However, a match does not form immediately upon meeting; both parties must agree to work together for the match to be established. Once a firm matches with a worker, production takes place. The production function for a match is:

$$y(k, h) = k^{1-\alpha} h^{\alpha},$$

where h is the human capital level and k the physical capital or capacity for this specific match. The firm also needs to incur sunk costs c per unit of capital when the match is formed. But it does not need to pay this cost if the match is not formed. To reach the agreement, both parties need to negotiate the wage paid to the worker and I assume it to be a fraction β of the output. Thus, the firm will get the rest $1 - \beta$. Again, this β can be understood as the bargaining power of worker side. In this static environment which is just like one period of game, both parties will get zero pay-off if they do not agree to form the match.

The expected value of a firm deciding on k in location i is then:

$$\begin{aligned} V_i(k, x^H, x^L) &= \phi_i x^H \left[(1-\beta) k^{1-\alpha} \eta^{\alpha} - ck \right] + (1 - \phi_i) x^L \left[(1-\beta) k^{1-\alpha} - ck \right] \\ &= \phi_i x^H (1-\beta) [k^{1-\alpha} \eta^{\alpha} - k] + (1 - \phi_i) x^L (1-\beta) [k^{1-\alpha} - k], \end{aligned} \tag{1}$$

where c is set to be $1-\beta$ for simplicity and x^j $(j = H,\ L)$ is the equilibrium probability that the firm hires the worker of j type.[1] I do not consider any mixed strategies, hence x^j being 0 or 1 and decided by the firm. The firms are not allowed to moved across locations. In each location, the firms decide on k, x^j to maximize (1) given the fraction of skilled workers, which partially determines the probability for them to meet one.

An equilibrium in this two location model contains the fractions of skilled workers ϕ_1 and ϕ_2 at which no workers will be better off by moving to other places, distribution of capital choices $F_i(k)$ over endogenously determined support K_i, and decision functions $x_i^H(k)$ and $x_i^L(k)$ such that for all $k \in K_i$, $(k, x_i^H(k), x_i^L(k)) \in arg\max\ V_i(k, x^H, x^L)$ for location i $(i = 1, 2)$.

In a partial equilibrium where ϕ_i is given, if $\eta < \left(\frac{1-\phi_i}{\phi_i^\alpha - \phi_i}\right)^{1/\alpha}$,all firms there will accept both types of workers, i.e., $x_i^H = x_i^L = 1$, and set capital $k_i^P = a[\phi_i \eta^\alpha + (1-\phi_i)]^{1/\alpha}$, where $a \equiv (1-\alpha)^{1/\alpha}$ for both types of workers. This is a pooling result. On the other hand, if $\eta \geq \left(\frac{1-\phi_i}{\phi_i^\alpha - \phi_i}\right)^{1/\alpha}$, the firms in location i will only hire skilled workers, i.e., $x_i^H = 1,\ x_i^L = 0$, and $k_i^H = a\eta$.[2]

To move from the partial equilibrium to general equilibrium for two locations, the main job is to endogenize the fractions of skilled workers in these two places under the assumption of free labor mobility. First, I denote a function for the threshold

$$\eta^T(\phi) = \left(\frac{1-\phi}{\phi^\alpha - \phi}\right)^{1/\alpha}.$$

As was discussed above, when the exogenous human capital difference $\eta$is lower than this threshold, there will be pooling results. It is easy to verify that η^T decreases with ϕ monotonically from 0 to 1. Moreover, $\eta^T \to \infty$ as $\phi \to 0$ and $\eta^T \to (1-\alpha)^{-1/\alpha}$ as $\phi \to 1$.

Different initial allocations of skilled workers, denoted as ϕ_1^o and ϕ_2^o , and the level of η will render different equilibrium results. In fact, there are multiple equilibria in many cases. Without specifying any rules or orders of workers moving, I just focus on two types of equilibrium: one for sorting of skilled workers in one place (the unskilled ones then agglomerate in the other) and the other for symmetric allocations. Before analyzing the general equilibrium, we still to define a way of how moving of a marginal worker can affect the fraction of skilled workers ϕ:

[1] To understand (1), since a firm meets one worker randomly, with probability of ϕ_i it will meet a skilled worker. Multiplying with the hiring probability x^H gives the probability of matching with a skilled worker $\phi_i x^H$. Then the firm will get $1-\beta$ of the total output while the capital cost has already been sunk.

[2] To derive this partial equilibrium, one can take F.O.C. of (1) with respect to k given ϕ_i, x_i^H and x_i^L. Then replace x_i^H and x_i^L with different values to calculate k and the values of $V_i(k)$ under different decision rules. Do the comparison and the conditions above will be obtained.

Definition 1. A large population economy is such that moving of one worker will not change the fractions of skilled workers in both places. In other words, a worker is of zero mass. And a small population economy is such that moving of one worker will change the fractions of skilled workers in both places.

The proposition below summarizes when these equilibria appear.

Proposition 1. *In this static model, if $\eta > (1-\alpha)^{-1/\alpha}$:*

a) The sorting of skilled workers to one place is always an equilibrium regardless of initial allocations of skilled workers.

b) The symmetric distribution can be an equilibrium only when i) $\eta > \max\{\eta^T(\phi_1^o),\ \eta^T(\phi_2^o)\}$ in a small population economy, and ii) $\eta > \min\{\eta^T(\phi_1^o),\ \eta^T(\phi_2^o)\}$ and $\eta < \max\{\eta^T(\phi_1^o),\ \eta^T(\phi_2^o)\}$ in a large population economy.

If $1 < \eta \leq (1-\alpha)^{-1/\alpha}$:

c) The sorting will not be an equilibrium regardless of initial allocations of skilled workers.

d) The symmetric allocation can be an equilibrium only when $\phi_1^o = \phi_2^o$ in a large population economy.

Proof. a) If $\phi_i = 0$ and $\phi_j = 1$, $\eta^T(\phi_i) = \infty > \eta$ and $\eta^T(\phi_j) = (1-\alpha)^{-1/\alpha} < \eta$. Then firms in place i will hire both types of workers and set the pooling capital as $k_i = a$ (setting ϕ_i to 0 for k_i^P as mentioned above) and pay $w_i^L = \beta a/(1-\alpha)$ to the unskilled and $w_i^H = \beta a\eta/(1-\alpha)$ to the skilled. And firms in place j will only hire the skilled and set $k_j = a\eta$ and pay $w_j^H = \beta a\eta/(1-\alpha)$ to them. The skilled and unskilled will only live in j and i respectively then. Since if a marginal skilled worker move to i in a small population economy, they will get a pooling wage at $\beta a[\phi_i\eta^\alpha + (1-\phi_i)]^{\frac{1-\alpha}{\alpha}}\eta^\alpha/(1-\alpha)$ for a small ϕ_i. And it is easy to verify that this wage level is lower than $\beta a\eta/(1-\alpha)$ when $\phi_i < 1$. Then no skilled workers will move. Neither do the low skilled workers since they will not even get hired. As for the case of large population economy, the skilled workers will get the unskilled pay-off by moving to the other place, hence no moving. Thus, this allocation is an equilibrium. And it does not depend on the initial worker allocations.

The rest of the proof is shown in Appendix A.1. □

Before discussing more on this result, let's look at the equilibrium in an otherwise Walrasian environment. The Walrasian allocation of this economy is such that firms and workers can switch partners without cost when bargaining over wage, and wage is the marginal product for each worker. It is easy to verify that the allocation of two types of workers is indeterminate while the skilled worker get $\alpha a\eta/[(1-\alpha)c^{\frac{1-\alpha}{\alpha}}]$ and the unskilled workers get $\alpha a/[(1-\alpha)c^{\frac{1-\alpha}{\alpha}}]$ in any place. The sorting will not necessarily happen.

Without any labor market frictions, the equilibrium outcome is simply symmetric, while agglomeration always emerges as an equilibrium when frictional labor markets are present in this setting. When firms cannot switch their worker partners at no cost and must make job capacity decisions before meeting workers, they face the risk of establishing a capital level without being able to find suitable matches for it. This risk is higher in locations where the proportion of skilled workers is small. In such places, firms are less inclined to invest in job positions and offer high wages. Conversely, if there are many skilled workers, firms become more willing to invest and even hire skilled workers exclusively. Meanwhile, workers can relocate to alter this proportion. Skilled workers can improve their income by increasing the proportion to a level at which firms will hire only them (wages for them in the separating equilibrium are always higher than in the pooling equilibrium). This creates a barrier for unskilled workers, leading to sorting. In contrast, the Walrasian market allocates skilled workers to high-capacity firms, maximizes output, and does not create barriers for unskilled workers.

Let's also examine the symmetric equilibria and their conditions. In the two symmetric equilibria that arise under different conditions, all firms hire only skilled workers. As a result, unskilled workers have no better options, as they are not paid anywhere. Skilled workers have no incentive to move since the wages in the separating equilibria are identical. Note that a large human capital difference, η, is necessary to achieve these results. The intuition is that when the skill gap between the two types of workers is large enough, all firms will take the risk of creating skilled job positions, as having a skilled worker makes a significant difference. In this case, the entire market effectively becomes homogeneous, leading to symmetry.

To summarize the intuition: in a frictional labor market, firms' hiring and investment decisions depend on the likelihood of meeting high-quality workers. A greater number of skilled workers in a location will increase firms' expected value of investing in jobs for those workers and hiring more of them instead of unskilled workers. These hiring decisions will then deter unskilled workers from entering areas where skilled workers agglomerate.

2.2 Dynamic model

The main results and intuition of the static model still hold in the dynamic version. To understand why a dynamic model is needed: in the agglomeration equilibrium of the static model, there are no unemployed workers, as they all move to locations that welcome them. Unemployment occurs only in the symmetric equilibria, where all unskilled workers are unemployed. These results are not sufficiently informative or helpful. Introducing labor matching frictions can help explain unemployment, and it is more effective in a dynamic setting. I extend the static model to include more general characteristics in a dynamic search model, such as endogenous job-finding and vacancy contact rates.

In a dynamic version, the timeline of the game should be specified in more detail. A firm enters the market and rents a site at an exogenous cost of γ. As in the static model, the firm decides on job capacity k and opens a job vacancy at that site before meeting a worker. A vacancy meets an unemployed worker at a rate of f_i, and an unemployed worker finds a vacancy at a rate of q_i with both rates endogenously determined by the unemployment and vacancy rates in the local market, as in the standard search model setting.[3]These rates are assumed to be negatively correlated, which becomes apparent when assuming a constant-returns-to-scale matching function. Once they meet and the worker's type is revealed, the firm decides whether to hire the worker. If the firm hires the worker, it incurs a sunk cost of ck, which does not apply to any other workers. If the firm and the worker reach an agreement during wage negotiation after the sunk cost has already been paid, they produce according to the output function specified in the previous section; otherwise, they continue searching for new partners. At a rate of s, the match dissolves, the worker becomes unemployed, the capital and site for the job become obsolete, and the firm exits.[4]

The value of a vacancy for the job of capital k, $J_i^V(k)$, satisfies

$$rJ_i^V(k, x^H, x^L) = -\gamma + q_i \left[\lambda_i x^H \left(J_i^H(k) - ck - J_i^V(k)\right) + (1-\lambda_i)x^L \left(J_i^L(k) - ck - J_i^V(k)\right)\right], \tag{2}$$

where λ_i is the equilibrium fraction of skilled ones among the unemployed workers in location i and r is the time discount rate. It says that the flow value of a vacancy equals to the expected pay-off from matching with a worker, who could be skilled and unskilled, after subtracting the site rental. The firms choose k, x^H and x^L to maximize $J_i^V(k)$ given q_i and λ_i. The asset value for a matched firm with capital k:

$$rJ_i^j(k) = k^{1-\alpha}h_j^{\alpha} - w_i^j(k) + s\left(J_i^V(k) - J_i^j(k)\right), \quad (j = H, L) \tag{3}$$

It says that the flow value of matching with a type j worker equals to the profits this match could generate and possibly getting separate next period.

The life-time utility is

$$\int_0^\infty e^{-rt} c_t \, dt,$$

[3]Notice that f_i and q_i are the same to different types of workers. Mortensen and Pissarides (1999) assume separating labor search market for different types of workers, hence different job finding and vacancy contact rates. I did not follow this since the separating labor market does not necessarily hold and it is clearer to illustrate the congestion through multiplying these location level terms with the fractions of skilled workers.

[4]A vacancy can be understood as a firm when the total output of a firm is of constant returns to scale, which means the size of firm does not matter.

where c_t is the consumption level at time t. The asset value for a matched worker of type j is then

$$rW_i^j(k) = w_i^j(k) + s\left(N_i^j - W_i^j(k)\right), \quad (j = H, L) \tag{4}$$

where the unemployed value of type-j worker in location i, N_i^j, satisfies

$$rN_i^j = b + f_i \int_{K_i} x_j(k)\left(W_i^j(k) - N_i^j\right) dF_i(k). \tag{5}$$

This equation says that the flow value of being unemployed equals to the unemployment benefits plus the expected gains from matching with a firm. With the distribution of firm investment choice $F_i(k)$, and the corresponding hiring decision $x^j(k)$ for type j worker, the expected gains are calculated as in the second term on the RHS of (5).

Following Acemoglu (1999), I let the wages be determined by bargaining with alternating offers rather than Nash bargaining which is usually used in the search literature. By doing so, the wages can simply be a fraction of output while the wages from Nash bargaining contain other terms like meeting rate and the separation rate. The wage setting is then:

$$w_i^j(k) = \max\left\{rN_i^j, \min\left[\beta k^{1-\alpha} h_j^\alpha, k^{1-\alpha} h_j^\alpha - rJ_i^j(k)\right]\right\}. \tag{6}$$

The steady state market clearing conditions:

$$u_i^j = \frac{s}{s + f_i \int_{K_i} x_i^j(k)\, dF_i(k)}, \tag{7}$$

$$\lambda_i = \frac{\phi_i u_i^H}{\phi_i u_i^H + (1-\phi_i) u_i^L}. \tag{8}$$

Free entry of firms:

$$J_i^V(k, x^H, x^L) = 0. \tag{9}$$

Free labor mobility says that the workers can go to any places they want. But that does not necessarily mean $N_1^j = N_2^j$ hold in the equilibrium. For example, if firms in one place only hire the skilled workers, it will be equivalent to restricting the mobility of unskilled workers. As was discussed in the first model section, that will give agglomeration.

The equilibrium contains functions $\{F_i(k), x_i^H(k), x_i^L(k)\}_{i=1,2}$, rates $\{\lambda_i, u_i^H, u_i^L, f_i, q_i, \phi_i\}_{i=1,2}$ such that market clearing conditions (3) to (9) are satisfied with (2) maximized and no workers will be better off by migration.

To solve for the equilibrium, the first step is to find the optimal capital level for different acceptance rules. Suppose bargaining does not result in corner solutions which is true after solving all the variables in the equilibrium. Then the value of a firm matching with a skill level j worker is:

$$J_i^j(k) = \frac{(1-\beta)k^{1-\alpha}h_j^{\alpha}}{r+s}.$$

Substitute it back to (2) and derive the F.O.C:

$$\lambda_i x^H[(1-\alpha)k^{-\alpha}\eta^{\alpha} - 1] + (1-\lambda_i)x^L[(1-\alpha)k^{-\alpha} - 1] = 0. \tag{10}$$

Next, take different values of x^H and x^L into (10) to get the optimal capital under different acceptance rules along with the vacancy value.

Under $x^H = x^L = 1$, a firm accepts both types of workers and posts a pooling job position with capacity $k_i^P = a(\lambda_i\eta^{\alpha} + 1 - \lambda_i)^{1/\alpha}$. The associated value of vacancy is

$$J_i^V(k^P) = \frac{1}{r+q_i}\left[-\gamma + \frac{q_i(1-\beta)\alpha a}{(r+s)(1-\alpha)}(\lambda_i\eta^{\alpha} + 1 - \lambda_i)^{1/\alpha}\right].$$

For $x^H = 1$ and $x^L = 0$, the firm only hires the skilled and posts the job position with capacity $k_i^H = a\eta$. The value of vacancy under this acceptance rule is:

$$J_i^V(k^H) = \frac{1}{r+q_i\lambda_i}\left[-\gamma + \frac{q_i(1-\beta)\alpha a\eta}{(r+s)(1-\alpha)}\right].$$

And it can be verified that if the firm only hire the unskilled ones, its vacancy value will be strictly less than the one of posting pooling job. Therefore, this strictly dominated strategy can be eliminated. I move on to compare the above two values.

The free entry condition implies that the maximum value of $J_i^V(k)$ is zero. Therefore, it will either be $J_i^V(k^P) = 0 > J_i^V(k^H)$ or $J_i^V(k^H) = 0 > J_i^V(k^P)$. Given q_i and λ_i, it can be verified that if $\eta > \eta^T(\lambda_i) = \left(\frac{1-\lambda_i}{\lambda_i^{\alpha}-\lambda_i}\right)^{1/\alpha}$, there will be $J_i^V(k^H) = 0 > J_i^V(k^P)$. And $J_i^V(k^P) = 0 > J_i^V(k^H)$ if $\eta \le \eta^T(\lambda_i)$. The threshold function is the same as the one in the static model. But the argument becomes the fraction of the skilled ones among the unemployed workers. Since the initial allocation again might matter in determining the equilibrium, I assume that in the beginning, $\lambda_i = \phi_i^o$, that is, all workers are unemployed.

The proposition on the symmetric equilibrium with sorting is stated as below:

Proposition 2. *In this dynamic model, if $\eta > (1-\alpha)^{-1/\alpha}$:*

a) The sorting of skilled workers to one place is always an equilibrium regardless of initial allocations of skilled workers.

b) The symmetric distribution can be an equilibrium only when i) $\eta > \max\{\eta^T(\phi_1^o),\ \eta^T(\phi_2^o)\}$ in a small population economy, and ii) $\eta > \min\{\eta^T(\phi_1^o),\ \eta^T(\phi_2^o)\}$ and $\eta < \max\{\eta^T(\phi_1^o),\ \eta^T(\phi_2^o)\}$

in a large population economy.

If $1 < \eta \leq (1-\alpha)^{-1/\alpha}$:

c) The sorting will not be an equilibrium regardless of initial distribution of skilled workers.

d) The symmetric allocation can be an equilibrium only when $\phi_1^o = \phi_2^o$ *in a large population economy.*

Proof. The proof of a), b), d) and e) is similar as that in Proposition 1. To see the unemployment rate differentials, one needs to use (3) and (9) to pin down the vacancy contact rate in two places. The vacancy contact rate in the highly skilled place is

$$q_H = \frac{\gamma(r+s)(1-\alpha)}{(1-\beta)\alpha a\eta},$$

and smaller than that in the unskilled area,

$$q_L = \frac{\gamma(r+s)(1-\alpha)}{(1-\beta)\alpha a}$$

Since the higher the vacancy contact rate, the lower job finding rate will be, i.e., $f_H > f_L$. The unemployment rate in the skilled area is then

$$u = u^H = \frac{s}{s+f_H},$$

which is smaller than

$$u_L = \frac{s}{s+f_L}.$$

□

The results and intuitions from the static model still apply here. In terms of unemployment rate differences within the sorting equilibrium, the mechanism remains centered around sorting. The value of matching with a worker is higher in areas where skilled workers are concentrated. As a result, firms move to those areas to open vacancies, which drives down the vacancy contact rate while increasing the job-finding rate. Consequently, these areas experience lower unemployment rates, as finding a job there becomes easier.

2.3 Prediction to be tested

Areas concentrated with skilled workers tend to have higher productivity and lower unemployment rates. According to the model, as different workers sort into different areas, the firms entering those areas also adjust their hiring and investment decisions accordingly. In regions with more skilled workers, firms invest more in each job, leading to higher productivity (output per worker). Wages in this model are proportional to output, resulting in higher average wages as well. The high output or return attracts more firms to areas with skilled workers, further increasing the job-finding rate and reducing the unemployment rate. This prediction captures the correlations among these variables rather than implying causal relationships, offering new insights into spatial differences in skilled worker distributions and unemployment rates.

2.4 Discussion

A weakness of the model stems from its strength: the simplicity of the equilibrium wage form, which results from bargaining with alternating offers. Under this bargaining rule, wages are proportional to match output. In contrast, if Nash bargaining were used, the wage form would include additional terms related to labor market tightness. By not using Nash bargaining, as many other search models do, my model cannot capture the congestion within skill groups. As more skilled workers move into one area, the effects of changes in market tightness on wages are not accounted for in the model.

In addition to the absence of the congestion effect from market tightness on wages, this model does not account for other forms of congestion, such as the classic housing rental costs discussed in urban economics literature. Incorporating congestion forces is essential to establish a unique equilibrium (see Allen and Arkolakis (2014)).

The producer side requires more structure. First, complementarity between different skill groups can be added to production, which could help capture the relationship between skill sorting and city size observed by Eeckhout et al. (2014). The purpose of linking sorting with size is to introduce agglomeration, a crucial factor in determining the size and activities across locations. Second, incorporating multiple industries is necessary to better model the relationship between the production structure and sorting. As I will explain below, there are empirical challenges in testing the second prediction, as the model does not address spillovers across industries. Finally, with a more complete producer-side structure, the model could incorporate trade, which is also essential for modeling economic geography.

3 Empirical evidence

3.1 Data

I mainly use 5%-sample Census data in 1990 and 2000 and American Community Survey (ACS) data from 2006 to 2019 to test the prediction. The empirical analysis focuses on the working-age group (16 - 64) and is conducted using commuting-zone level observations.The job finding and separation rates for each commuting zone are measured indirectly, as neither the 5%-sample Census nor the ACS provide explicit information on individuals' lagged employment statuses. [5] However, both datasets include a question regarding the number of weeks a respondent worked in the previous year, with responses categorized into intervals such as 0, 1-13, 14-26, 27-39, 39-47, and so on. I classify workers as employed if they worked 26 weeks or more in the previous year. I cross-validate this measure using various data sources and find it to be highly correlated with them (see Appendix A.2 for further details). I define workers as unemployed if they worked fewer than 26 weeks in the previous year but are still participating in the labor market in the current year. Employment transition rates are calculated annually. If an individual was employed last year but becomes unemployed this year, they are counted as having experienced job separation. Conversely, if an individual was unemployed last year but is employed this year, they are counted as having found a job. I restrict the survey sample to the working-age population, defined as individuals aged 16-64, and arrange the variables for 741 commuting zones in the U.S. for the years 1990, 2000, and 2006-2019.

3.2 Regional employment patterns

The first empirical exercise tests the predicted regional employment patterns: regions with a higher fraction of skilled workers are expected to have higher labor productivity, lower unemployment rates, and higher job-finding rates. Since these variables are determined in equilibrium, my goal is not to establish causal identification but to examine the correlations among them in the data. To clarify the measurement, I define a skilled worker as anyone currently in the labor force, whether employed or not, who has completed at least four years of college education (e.g., a master's degree). While this is not an explanatory variable in a causal analysis, I use it as the main independent variable to illustrate the correlation. I calculate the fraction of skilled workers in a commuting zone using 5%-sample Census data in 1990 and 2000, along with ACS data from 2006 to 2019, applying the weights assigned to

[5] CPS tracks the employment statuses of respondents but does not have geographic information at the commuting zone level.

each survey participant by the Census.

The average wage and salary income in a region serves as a proxy for productivity in that region. The rationale is that labor productivity in the model represents the output produced by a worker-firm match, and the wage is proportional to this output. Similarly, the average wage and unemployment rate are calculated using Census/ACS data. Job finding rates are calculated according to the method described above. I regress the regional unemployment rate, average wage and job finding rate on the fraction of skilled workers in the regional labor force to test the prediction:

$$y_{rt} = \alpha_0 + \alpha_1 SkilledRate_{rt} + \lambda_t + \lambda_r + \epsilon_{rt}, \tag{11}$$

where y_{rt} is the outcome variables in CZ r in year t, and λ_t and λ_r are year and CZ fixed effects. Region and time fixed effects are controlled to exclude any region-specific shocks or aggregate national shocks that can help generate regional disparities.

I test the prediction using the year 2000 samples first. As is shown in Table 1, empirical results support the prediction. Higher skilled worker fraction is positively associated with regional average wage and job finding rate, and negatively with unemployment rate. In the data, increasing the skilled worker fraction from 25th to 75th percentile is found to be equivalent to increasing the skilled worker fraction by around 6% in each year. According to the table, that suggests moving from an area at 25th percentile of skilled worker fraction to 75th percentile is associated with a decrease of about 1 percentage point in unemployment, which is of a large magnitude given that CZ unemployment rate is averagely 5%. It is also associated with 0.1% increase in average wage and 2 percentage points increase in job finding rate.

Although some of these patterns can be explained by other existing models, there are still valuable empirical facts that enhance our understanding of employment. The positive correlation between a higher fraction of skilled workers and average wages can be attributed to the skill wage premium, while the finding that it also raises the job-finding rate and lowers the unemployment rate is novel. According to the model, firms will relocate to areas where skilled workers agglomerate, resulting in more job opportunities and, consequently, a higher job-finding rate. Without this firm sorting, the crowding of skilled workers in one place would not lead to a higher job-finding rate.

Table 1: CZ-level regression results in 2000

	(1) Unemp. Rate	(2) log(Wage)	(3) Job Find. Rate
Skilled Rate	-0.154***	0.016***	0.346***
	(0.013)	(0.001)	(0.035)
Constant	8.613***	9.947***	69.797***
	(0.252)	(0.014)	(0.661)
Observations	741	741	741

Notes: Results are estimated using regression 11, excluding the fixed effects. Data are from 5

I further test the prediction using samples from all years that I obtain, with CZ and year fixed effects controlled. The results are robust as shown in Table 2. I also use different measures of job finding rates to test the prediction and find them to be robust, as shown in Table 7 in Appendix.

Table 2: CZ-level regression results across years

	(1) Unemp. Rate	(2) log(Wage)	(3) Job Find. Rate
Skilled Rate	-0.079***	0.009***	0.248***
	(0.009)	(0.000)	(0.043)
Constant	8.394***	10.306***	56.121***
	(0.181)	(0.006)	(0.815)
Fixed effect	CZ, Year	CZ, Year	CZ, Year

Notes: Results are estimated using regression 11. Data are from 5

4 Conclusion

This paper aims to link spatial sorting with spatial differences in productivity, wages, and unemployment through a new channel: the frictional labor market. It seeks to answer the question: how can the frictional labor market explain spatial sorting and, consequently, disparities?

I first demonstrated the main mechanism by which a frictional labor market generates spatial labor sorting through a static search model with two locations and free labor mobility.

Spatial sorting, characterized by the segregation of skilled and unskilled workers, occurs when the human capital difference between these two types of workers is sufficiently large. The intuition is that in a frictional labor market, firms' hiring and investment decisions depend on the likelihood of meeting high-quality workers. An increase in skilled workers in a given area raises firms' expected value of investing in jobs for those workers and hiring more skilled rather than unskilled workers. As a result, hiring decisions discourage unskilled workers from entering areas where skilled workers agglomerate.

I extended the static model to a dynamic one to incorporate more general features of a search model that can address unemployment. The main results and intuitions from the static model still hold, with the dynamic model predicting that areas where skilled workers sort will have lower unemployment rates. This is because, according to the model, these areas attract more firms seeking higher profits, which increases the job-finding rate and, consequently, lowers the unemployment rate.

The model further predicts that the places concentrated with skilled workers tend to have higher productivity and lower unemployment rate. I test the prediction at the commuting zone level using Census/ACS data. The results indicate that a higher fraction of skilled workers is positively associated with regional average wages and job-finding rates, and negatively associated with unemployment rates. The job finding rates for each commuting zone are measured indirectly, as neither the 5%-sample Census nor the ACS provide explicit information on individuals' lagged employment statuses. I exploit the answers to a question from these survey data to obtain proxies for the individuals' lagged employment statuses.

Future research will focus on extending the model to: i) incorporate the congestion effect by using Nash bargaining, allowing market tightness to influence wages, as well as other forms of congestion, such as housing rental costs; ii) add complementarity between different skill groups in production, linking sorting to city size in line with empirical patterns found in existing literature; iii) include multiple industries to understand inter-industry spillovers from changes in the production structure, leading to more precise empirical implications; and iv) introduce trade between firms and locations to better model economic activities across space.

Bibliography

Acemoglu, Daron. 1999. "Changes in unemployment and wage inequality: An alternative theory and some evidence." *American economic review* 89 (5): 1259–1278.

Acemoglu, Daron, and David Autor. 2011. "Skills, tasks and technologies: Implications for employment and earnings." In *Handbook of labor economics*, Volume 4. 1043–1171, Elsevier.

Allen, Treb, and Costas Arkolakis. 2014. "Trade and the Topography of the Spatial Economy." *The Quarterly Journal of Economics* 129 (3): 1085–1140.

Andersson, Fredrik, Simon Burgess, and Julia I Lane. 2007. "Cities, matching and the productivity gains of agglomeration." *Journal of Urban Economics* 61 (1): 112–128.

Autor, David H, and David Dorn. 2013. "The growth of low-skill service jobs and the polarization of the US labor market." *American economic review* 103 (5): 1553–1597.

Behrens, Kristian, Gilles Duranton, and Frederic Robert-Nicoud. 2014. "Productive cities: Sorting, selection, and agglomeration." *Journal of Political Economy* 122 (3): 507–553.

Behrens, Kristian, and Frederic Robert-Nicoud. 2015. "Agglomeration theory with heterogeneous agents." *Handbook of regional and urban economics* 5 171–245.

Bilal, Adrien. 2023. "The geography of unemployment." *The Quarterly Journal of Economics* 138 (3): 1507–1576.

Combes, Pierre-Philippe, Gilles Duranton, and Laurent Gobillon. 2008. "Spatial wage disparities: Sorting matters!." *Journal of urban economics* 63 (2): 723–742.

Combes, Pierre-Philippe, and Laurent Gobillon. 2015. "The empirics of agglomeration economies." In *Handbook of regional and urban economics*, Volume 5. 247–348, Elsevier.

Davis, Donald R, and Jonathan I Dingel. 2019. "A spatial knowledge economy." *American Economic Review* 109 (1): 153–170.

Davis, Donald R, and Jonathan I Dingel. 2020. "The comparative advantage of cities." *Journal of International Economics* 123 103291.

Diamond, Rebecca. 2016. "The determinants and welfare implications of US workers' diverging location choices by skill: 1980–2000." *American Economic Review* 106 (3): 479–524.

Eeckhout, Jan, Roberto Pinheiro, and Kurt Schmidheiny. 2014. “Spatial sorting.” *Journal of Political Economy* 122 (3): 554–620.

Ehrlich, Maximilian v, and Henry G Overman. 2020. “Place-based policies and spatial disparities across European cities.” *Journal of economic perspectives* 34 (3): 128–149.

Kline, Patrick, and Enrico Moretti. 2013. “Place based policies with unemployment.” *American Economic Review* 103 (3): 238–243.

Marinescu, Ioana, and Roland Rathelot. 2018. “Mismatch unemployment and the geography of job search.” *American Economic Journal: Macroeconomics* 10 (3): 42–70.

Matano, Alessia, and Paolo Naticchioni. 2012. “Wage distribution and the spatial sorting of workers.” *Journal of Economic Geography* 12 (2): 379–408.

Mion, Giordano, and Paolo Naticchioni. 2009. “The spatial sorting and matching of skills and firms.” *Canadian Journal of Economics/Revue canadienne d’économique* 42 (1): 28–55.

Moretti, Enrico. 2013. “Real wage inequality.” *American Economic Journal: Applied Economics* 5 (1): 65–103.

OECD. 2005. “How persistent are regional disparities in employment? The role of geographic mobility.” In *OECD Employment Outlook 2005*, 73–123, OECD Publishing Paris.

Tabuchi, Takatoshi, Jacques-François Thisse, and Xiwei Zhu. 2018. “Does technological progress magnify regional disparities?” *International Economic Review* 59 (2): 647–663.

A Appendix

A.1 Proposition proof

i) $\eta > (1-\alpha)^{-1/\alpha}$:

When $\phi_i = 0 and \phi_j = 1, \eta^T(\phi_i) = \infty > \eta$ and $\eta^T(\phi_j) = (1-\alpha)^{-1/\alpha} < \eta$. Then the firms in place i will hire both types of workers, set the pooling capital as $k_i = a$ (setting ϕ_i to 0 for k_i^P mentioned above), and pay $w_i^L = \frac{\beta a}{1-\alpha}$ to the unskilled workers, and $w_i^H = \frac{\beta a \eta}{1-\alpha}$ to the skilled workers. The firms in place j will only hire skilled workers, set $k_j = a\eta$, and pay $w_j^H = \frac{\beta a \eta}{1-\alpha}$ to them. Thus, skilled workers will only live in j, while unskilled workers will live in i. If a marginal skilled worker moves to i in a small population economy, they will receive a pooling wage at:

$$\beta a \left[\varphi_i \eta^\alpha + (1-\varphi_i)\right]^{\frac{1-\alpha}{\alpha}} \frac{\eta^\alpha}{1-\alpha}$$

for a tiny ϕ_i. It is easy to verify that this wage level is lower than $\frac{\beta a \eta}{1-\alpha}$ if $\phi_i < 1$. Therefore, no skilled workers will move. The unskilled workers will not move either, since they will not be hired. In the case of a large population economy, the skilled workers would receive the unskilled payoff by moving to the other place, hence no movement occurs. Thus, this allocation is an equilibrium and does not depend on the initial worker allocations. To determine when a symmetric allocation appears, we need to consider different situations:

Small population economy:

1. $\eta > \max\{\eta_T(\phi_1^o), \eta_T(\phi_2^o)\}$: Firms in both places will hire only skilled workers, resulting in a symmetric allocation of both worker types. Skilled workers receive the same wages in both places, while unskilled workers receive zero pay regardless of location.

2. $\min\{\eta_T(\phi_1^o), \eta_T(\phi_2^o)\} \le \eta \le \max\{\eta_T(\phi_1^o), \eta_T(\phi_2^o)\}$: Assume $\eta_T(\phi_1^o) = \min\{\eta_T(\phi_1^o), \eta_T(\phi_2^o)\}$ and $\eta_T(\phi_2^o) = \max\{\eta_T(\phi_1^o), \eta_T(\phi_2^o)\}$. The symmetric allocation will not hold, as skilled workers will move to place 1, where only skilled workers are hired, and unskilled workers will remain in place 2.

3. $\eta < \min\{\eta_T(\phi_1^o), \eta_T(\phi_2^o)\}$: Firms in both places will offer pooling positions and pay pooling wages. Since pooling wages increase with the fraction of skilled workers, symmetric allocation will not hold in equilibrium; people will move to alter the skilled fraction and improve their income.

Large population economy: the first two are exactly the same as the small population economy. For the last point, since now the skilled fraction will not be changed by moving of a worker, the symmetric allocation can hold as an equilibrium.

ii) $1 < \eta \leq (1-\alpha)^{-1/\alpha}$:

When $\phi_i = 0$ and $\phi_j = 1$, $\eta^T(\phi_i) = \infty > \eta$ and $\eta^T(\phi_j) = (1-\alpha)^{-1/\alpha} \geq \eta$. Firms in both places offer pooling jobs and wages. Workers in the area with $\phi_i = 0$ would benefit by moving to the other area, as pooling wages increase with ϕ. This behavior is independent of the initial allocations.

In a small population economy, workers will always move, even if both places start with the same skilled worker allocation, as they can change the fraction to affect wages. In a large population economy, when two places start with the same initial fraction of skilled workers, workers will not move, as they cannot change the fractions. They will stay put and accept the same wage in both places.

A.2 Cross-validation of job transition measures

The employment status for an agent in the previous year is identified according to the answer to a question asking how many weeks the agent worked for last year. The answers are categorized into the following intervals (see WKSWORK2 in the IPUMS ACS): N/A or missing, 1-13 weeks, 14-26 weeks, 27-39 weeks, 40-47 weeks, 48-49 weeks, 50-52 weeks. Denote a threshold of weeks to be T ($T = 13, 26, 39, 47$). I define an agent to be unemployed last year if they worked for fewer than T weeks and they are in the labor force this year. I assume people who are in the labor force this year were also in the labor force last year. And an agent is counted as being employed last year if they worked for more than T weeks.

The first cross-validation is to use ACS dataset itself and do the commuting-zone level calculation and validation. For each commuting zone, I can calculate the employment and unemployment in year $t-1$ using ACS data of year t. Meanwhile, I can obtain the employment and unemployment in year $t-1$ directly using the employment status information in ACS data of year $t-1$. Since ACS has continuous samples with puma code (a code that can be used to identify CZ) from 2006 and onward, I do the validation for all CZ across 2006 to 2018, which covers the whole period used in the empirical part. It turns out that all these four thresholds offer high correlations.The following two tables summarize the correlation between the data moments measured using different thresholds and the actual data moments.

Table 3: Correlation between the CZ unemployment calculated using different thresholds and those from ACS

Variable	L^U_{13wks}	L^U_{26wks}	L^U_{39wks}	L^U_{47wks}
Corr.	0.9487	0.9521	0.9543	0.9545

Table 4: Correlation between the CZ employment calculated using different thresholds and those from ACS

Variable	L^E_{13wks}	L^E_{26wks}	L^E_{26wks}	L^E_{26wks}
Corr.	0.9736	0.9741	0.9747	0.9747

Next, I use CPS data to do the cross-validation. CPS is essentially a short panel dataset, tracking agents for about one year. However, it only has accurate records of geographic information at the state level[6]. Another concern is that there were many respondents who dropped out of sample and could not be tracked. Therefore, I calculate the job transition

[6] It has metropolitan information but there are too many missing observations to be used.

rates using gross flow ratios at the state level. For example, I obtain the total number of the unemployed who became employed after a year UE_t, and the total number of the unemployed at the beginning of the year U_t. The job finding rate is then UE_t/U_t. I compare the state-level job transition rates I construct using those four thresholds in ACS data with the job transition rates calculated using CPS data. The correlations for job finding rates increase with the threshold while the correlations for job separation rates decrease with it. The results are shown in the following two tables:

Table 5: Correlation between the state-level job finding rates calculated using different thresholds and those calculated from CPS

Year	JF_{13wks}	JF_{26wks}	JF_{39wks}	JF_{47wks}
1990	0.78	0.79	0.82	0.80
2000	0.71	0.71	0.72	0.74
2007	0.81	0.82	0.83	0.87

Table 6: Correlation between the state-level job separation rates calculated using different thresholds and those calculated from CPS

Year	JS_{13wks}	JS_{26wks}	JS_{39wks}	JS_{47wks}
1990	0.74	0.69	0.61	0.56
2000	0.66	0.56	0.50	0.45
2007	0.59	0.57	0.56	0.48

According to all these results, I choose 26 weeks as the threshold.

Table 7: Different thresholds for job finding rates

	(1)	(2)	(3)
	JF_13wks	JF_39wks	JF_47wks
Skilled Rate	0.152***	0.288***	0.321***
	(0.053)	(0.035)	(0.030)
Constant	47.791***	62.613***	67.114***
	(1.016)	(0.663)	(0.573)
Fixed effect	CZ, Year	CZ, Year	CZ, Year

Notes: *p<0.1; **p<0.05; ***p<0.01.